doi:10.6041/j.issn.1000-1298.2016.07.026

Mathematical Model of Soil Nutrient along Surface Runoff under Water Scouring Condition

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Abstract: Overland flow is one of the important factors inducing soil nutrients loss into runoff. In order to simulate and study the effects of overland flow on soil nutrients loss, a mathematical model describing soil nutrients transport with overland flow was developed based on the concept of effective mixing depth and Kostiakvo equation. In addition, the model also makes the corresponding assumptions to simplify the infiltration process. The obtained model was denoted as the incomplete-mixing model. Field water scouring experimental data was adopted to test the applicability of the proposed model. The experiment was performed on a natural, fallowed loessial slope (38°46′ ~ 38°51′N, 110°21′ ~ 110°23′E), in 14 km west of the Shenmu Erosion and Environment Research Station of the Institute of Soil and Water Conservation, Chinese Academy of Sciences, in the Liudaogou watershed, Shaanxi Province. Field plots were established on a slope that had been fallowed for seven years. Vegetation types cover were adopted (herbaceous and crops), including caragana and soybean. The characteristics of the transport of runoff, nutrients (nitrogen and phosphorus) under different vegetation cover were discussed in this study. Results indicated that the incomplete-mixing model performed pretty well in predicting the process of nutrients transport into runoff. The correlation coefficients were larger than 0.9 for all the treatments. Moreover, little bias was observed between the measured cumulative mass and the simulated data obtained from the incomplete-mixing model. Taking nitrate nitrogen for example, the relative errors between the measured data and simulated results were 6.6% (caragana) and 5.9% (soybean). When the nutrient was soluble phosphorus, the relative errors were 1.1% (caragana) and 2.3% (soybean). For better simulation results, the calculation accuracy and simplicity in obtaining parameters should be taken into consideration during model selection. The results in this study will provide significant references for more analysis on nutrients transport into runoff with overland flow in future.

Key words: runoff scouring; loess slope; effective mixing depth; nutrient transport; surface runoff

0 Introduction

The overland flow on a hillslope constitutes the main cause of rill formation and nutrients loss. With the loss of soil nutrients, the production capacity of the soil reduced. Furthermore, nutrients and soil transfer with the overland flow silt up in the rivers and lakes, leading to the eutrophication of the water body. In the loess plateau of China, water and soil erosion is more serious[1-3]. Scholars from home and abroad have made a lot of research and put forward various types of mathematical models to describe soil nutrient losses into runoff[4-7]. At the beginning of nutrients models development, the mixing layer was the most commonly used for modeling chemical transport to the runoff[8-9]. Early in 1977, DONIGIAN, et al.[10] proposed a model stimulating the soil nutrient transport with the runoff. It assumed that chemical transfer from a mixing zone in which rainwater, soil water, and infiltrating water mix instantaneously, completely, and uniformly, with no chemical exchange from below the mixing zone. After that, the dissolved nutrients in the mixing layer transfered with the infiltration and runoff water. The thickness of the mixed layer was obtained by curve fitting the experimental data. FRERE, et al.[11] assumed that the depth of the mixed layer was 1.0 cm. However INGRAM, et al.[12] found that the concentration of the nutrient in runoff was far lower than that in soil. So they proposed the incomplete-mixing concept and considered that the nutrient

Received date: 2016-03-10 Accepted date: 2016-04-19
Supported by National Natural Science Foundation of China (Key Program) (Grant No. 51239009)
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concentration in soil was the same with that of infiltration, and in proportion to the concentration of runoff. United States Department of Agriculture developed the non-point source pollution model namely CREAMS model on the basis of the "mixed layer" concept in 1980. AHUJA, et al.[13] used ^{32}P as the tracer material and found that the interaction between rainwater and soil water decreased rapidly with depth under free infiltration and saturated water conditions. So they proposed the concept of effective mixing depth (EDI), and EDI is a generalized thickness, rather than the actual thickness of the soil. The complete-mixing model was then proposed, it hypothesed that rainwater completely mixes with soil water in the effective mixing layer, and the solute concentration in the runoff equalled to the concentration in the infiltrating water and in the mixing layer. Besides, through the experimental study, AHUJA, et al.[14-15] found that under the influence of large infiltration rate, the logarithm value of runoff nutrient concentration and the time was a curve relationship. It showed that the complete-mixing model was not reasonable for the soil with large infiltration capacity. ZHANG[16] studied the mineral nitrogen and the runoff depth, and put forward the backstepping method and the turning point method to calculate EDI. According to soil nutrient transport equation, WALLACH, et al.[17], obtained the distribution of soil nutrient profile, through the analysis of the saturated soil nutrient transport characteristics on the undrained case and they considered that the soil nutrient had entered the runoff through mass transfer mode. So they proposed the concept of effective transmission depth (EDT). According to the specific characteristics of the loess area, WANG[18] proposed the concept of the equivalent runoff migration depth. He divided the effective mixing depth into two parts. One part is equivalent infiltration depth, and the other is the equivalent runoff migration depth. They established the equivalent runoff migration depth solving equation, and according to the experimental data, he deduced the equivalent runoff migration depth of K^+ under different runoff equivalent raindrop kinetic energy. Because of the serious soil and water loss in the loess area, the index model established by the majority of foreign scholars did not take into account the loss of soil and water, so according to a large number of experimental data. WANG, et al.[19] proposed the equivalent convective mass transfer model, and the simulation results were in agreement with the experimental results. After that , WANG, et al.[20] with the effective mixing depth concept and Philip infiltration equation[21], deduced the effective mixing depth model of the soil nutrients migrating with the runoff under the condition of rainfall. YANG, et al.[22] supposed that the mixing depth is not a constant in the nutrients transport model, and established a relationship of mixing depth among various influencing factors, such as rainfall intensity, slope, initial water content and so on by using regression analysis method.

At present, the propsoed nutrients transport models are mainly in accordance with the condition of rainfall, and the model development is relatively mature. But we are now lacking of the mathematical models that soil nutrient transport with runoff under the runoff scouring condition. So the description of the interaction between the rain and runoff erosion in the nutrient transport is limited. Therefore, it is needed to analyze the effect of water scouring on the surface runoff of soil nutrients. Based on runoff infiltration process of generalization, combined with Kosjakov infiltration formula and the mixing depth model of nutrient transfer, a mathematical model was established to study the effect of water flow erosion on the surface runoff of the slope surface, which provides the basis for the development of the mathematical model of the soil nutrients with the effect of rain and runoff transport with the runoff.

1 Materials and methods

1.1 Test equipment and materials

The experiment was performed on a natural, fallowed loessial slope (38°46′~38°51′N, 110°21′~110°23′E), in 14 km west of the Shenmu Erosion and Environment Research Station of the Institute of Soil and Water Conservation, Chinese Academy of Sciences, in the Liudaogou Watershed, Shaanxi Province at Shenmu. Water and soil erosion is serious and such natural disasters as rainstorm, drought, etc. are heavy in this area. The soil type is sandy loess, including 15.19% clay, 36.90% silt and 47.91% sand respectively (mass fraction). The soil bulk density of surface soil within 50 cm is in the range of 1.26~1.41 g/cm^3, and the average bulk density is

1.34 g/cm^3. A total of four experiment plots are set, installing baffles along the slope direction on both sides of plot to prevent water outflow, and install V type trough to collect runoff in outlet. Slope vegetations are Caragana and soybean, each has a repeat and were planted with 30 cm row spacing. Before runoff scouring experiment, we use leveling instrument to level all slope plots and ensure slope surface water flow at the same time to avoid plume. A steady-head water scouring method was adopted under natural conditions and water supply equipment is consist of the cellar, movable type reservoir, water pumps and generators. Test schematic diagram is shown in Fig. 1.

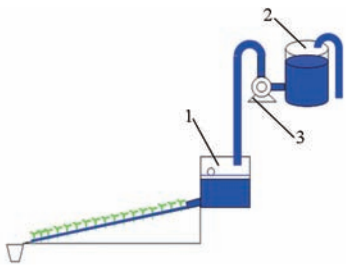

Fig. 1 Schematic diagram of water scouring experiment
1. Reservoir 2. Cellar 3. Water pumps

1.2 Test method

The nutrient spraying standard is KNO_3 for 60 g/m^2, KH_2PO_4 for 60 g/m^2 according to the level of fertilizer used in Shenmu. After spraying, placing 24 h, so that the nutrients can have enough time to distribute in the surface layer of a certain depth.

The test plot is 10 m in length, 1m in width, and the slope is 10.8°. The upslope runoff flow is 21 L/min (the discharge of water of per unit area is 0.21 cm/min), and the flow needs to do multiple times of rating before the test. When the value anduniformity of the flow meet the requirements, start the flow erosion test. The time of single field discharge is 40 min. After the beginning of the flow erosion test, record the time that the overland flow gets to the section which was set up already before the test, and make this time as the determination of water flow propulsion. When the overland flow reached to the outlet, the runoff production time is recorded. After the runoff occurs, a plastic bucket with a capacity of approximately 18 L is used to collect the runoff samples per 1min, and measured the quality of each water sample of runoff (convert to volume when dealing with the data). Then air-dry the sediment in each runoff barrel to determine the sediment quality of each period of time. When the overland flow is stable, use staining method (Potassium Permanganate solution) to measure the flow rate.

Collect the supernatant of the runoff from the plastic buckets which are used to collect the runoff with the capacity of 100mL plastic bottles, and keep it in cold storage. Then measure the concentration of the nitrate and water soluble phosphorus. Before and after each water discharging test, collect soil profile samples from the top, middle and bottom of the slope direction to measure the soil nutrient concentration. Collect the surface 10 cm soil in accordance with the principle of 0 ~ 1 cm, 1 ~ 3 cm, 3 ~ 5 cm, 5 ~ 7.5 cm, 7.5 ~ 10 cm, and the soil samples under 10cm are collected once every 5 cm and the sampling depth is 50 cm. When mensurating the soil profile moisture content, the soil sampling principle is the same as the above-metioned, and the moisture content of soil is determined by drying method. After taking soil sample and measuring soil moisture, use the soil near the plot to backfill in the same quality, and the compaction is carried out, so that the density of the filled soil is close to the original soil in the area. The soil saturated water rate of the experimental plot is 40.55% (volumetric water rate), and the initial water rate are 9.575% (Caragana) and 10.67% (Soybean) (both of them are volumetric water rate). The concentration of the nitrate nitrogen and water soluble phosphorus is analyzed by UV spectrophotometer, and the soil available phosphorus is analyzed by 0.5 mol/L $NaHCO_3$ leaching molybdenum antimony anti colorimetric. Take three times repetition of each sample and make the arithmetic mean value as the calculating and analyzing result. Use this method to measure the initial contents of the nitrate nitrogen and water soluble phosphorus of the surface soil, and they are 339.12 mg/kg and 451.05 mg/kg, respectively. The isothermal linear absorption coefficient of nitrate nitrogen and available phosphorus in soil is 0.83 L/kg and 2.1 L/kg determined by the experiment. Then use Matlab and Excel software to process data.

1.3 Theory

The Kostiakov equation to describe the slope infiltration process under the conditions of water erosion. For the slope with short length, in order to simplify the infiltration process, consider the slope as a unit, and assumed that the average infiltration time is $t_p/2$. Therefore, the Kostiakov equation can be expressed as

$$i = a\left(t - \frac{t_p}{2}\right)^{-b} \tag{1}$$

$$I = \frac{a}{1-b}\left(t - \frac{t_p}{2}\right)^{1-b} \tag{2}$$

In Eqs. (1) and (2), i is infiltration rate (cm/min); I is cumulative infiltration volume (cm); t is discharge time (min); t_p is runoff production time (min); a is the soil infiltration rate at the end of the first unit of time (cm/min); b is empirical index.

The process of soil nutrient transport can be divided into two parts, one is with the water seepage to the deep migration, and the other is the surface runoff. Because the traditional mixing depth model is mostly based on saturated soil and rainfall conditions, therefore, in order to establish the mathematical model for the transfer of unsaturated soil nutrients to surface runoff under the condition of water erosion, the process of soil moisture and nutrient transport is generalized. It is assumed that the overland flow makes the mixing depth reach saturation firstly, and then the soil moisture passes through the mixing layer, which carries nutrients to the deep layer. The nutrients in the effective mixing depth are always evenly distributed, and the probability of entering the surface runoff is the same. Regardless of the way in which nutrients enter the surface runoff, it will be considered as a dissolved state into the runoff, and consider the comprehensive effect of soil erosion as the influence to the effective mixing depth and concentration ratio coefficient. Nutrient concentrations in the mixed layer during the whole discharge period and the nutrient concentration in the soil and the nutrient concentration in the surface runoff are fixed proportion[20].

If the saturated water content of soil is θ_s (cm^3/ cm^3), the initial water content is θ_i (cm^3/cm^3), and the effective mixed layer depth is h_m (cm), then the water needed in the mixing depth to saturation is:

$$w_0 = (\theta_s - \theta_i)h_m \tag{3}$$

$$I_0 = w_0 \tag{4}$$

$$\frac{a}{1-b}\left(t_0 - \frac{t_p}{2}\right)^{1-b} = (\theta_s - \theta_i)h_m \tag{5}$$

The time required for soil t_0 required for soil at the saturated effective mixing depth is

$$t_0 = \left[\frac{1-b}{a}(\theta_s - \theta_i)h_m\right]^{\frac{1}{1-b}} + \frac{t_p}{2} \tag{6}$$

In Eqs. (4), (5) and (6), I_0 is the cumulative infiltration at the time of t_0 (cm). If $t_p < t_0$, adjust the effective mixing depth to make them equal. If $t_p > t_0$, calculate the nutrient concentration in the mixing layer according to the above hypothesis. If the nutrient adsorption process is considered as the isothermal adsorption process, it can be expressed as

$$c_s = kc_0 \tag{7}$$

In Eq. (7), c_s is the nutrient component that adsorbed to the soil particles (mg/kg); k is isothermal linear adsorption coefficient (L/kg); c_0 is the nutrient concentration of soil solution (mg/L).

According to the effective mixing depth concept proposed by AHUJA[4], the nutrient concentration in the effective mixing depth, the nutrient concentration in the water and the nutrient concentration in runoff are in proportion. It is assumed that the mixed layer begins to deliver nutrients to the surface at the time t_p, then the nutrient concentration in effective mixed depth can be expressed as

$$c_m = \frac{h_m(\theta_s + \rho_s k)c_i}{\alpha\left[\frac{a}{1-b}\left(\frac{t_p}{2}\right)^{1-b} - (\theta_s - \theta_i)h_m\right] + h_m(\theta_s + \rho_s k)} \tag{8}$$

In Eq. (8), ρ_s is bulk density of soil (g/cm^3); α is the ratio of nutrient concentration in the infiltration and the effective mixing depth; c_i is the nutrient concentration that the initial concentration of soil nutrients is converted into the saturated state.

After runoff, one part of the nutrient in the effective mixing depth migrats with the runoff, the other gets into the deep migration with the infiltration. So the nutrients in the effective mixing depth changing with the time can be expressed as

$$\frac{dch_m(\theta_s + \rho_s k)}{dt} = -\alpha ic - \beta qc \tag{9}$$

In Eq. (9), β is the ratio of nutrient concentration in runoff and effective mixed depth; q is runoff intensity of per unit area (cm/min); c is the nutrient concentration in effective mixed depth at any time after runoff (mg/L).

The following formula is got.

$$c = c_m \exp\left(- \frac{\alpha \int_{t_p}^{t} i \mathrm{d}t + \beta \int_{t_p}^{t} q \mathrm{d}t}{h_m(\theta_s + \rho_s k)} \right) \tag{10}$$

Runoff nutrient concentration is expressed as

$$\beta c = \beta c_m \exp\left(- \frac{\alpha \int_{t_p}^{t} i \mathrm{d}t + \beta \int_{t_p}^{t} q \mathrm{d}t}{h_m(\theta_s + \rho_s k)} \right) \tag{11}$$

In Eq. (11), the first integral term represents the accumulated in infiltration and the second integral term represents the cumulative runoff. The formula of Kostiakov is used to calculate the cumulative infiltration, and the cumulative runoff is obtained by the total inflow subtracting the total amount of infiltration. It can be expressed as

$$\beta c = \beta c_m \exp\left(- \frac{\alpha \frac{a}{1-b}\left[\left(t - \frac{t_p}{2}\right)^{1-b} - \left(\frac{t_p}{2}\right)^{1-b} \right] + \beta q_0 (t - t_p) - \beta \frac{a}{1-b}\left[\left(t - \frac{t_p}{2}\right)^{1-b} - \left(\frac{t_p}{2}\right)^{1-b} \right]}{h_m(\theta_s + \rho_s k)} \right) \tag{12}$$

In Eq. (12), q_0 is the rate of inflow (cm/min). Further simplified, the incomplete-mixing model is obtained.

$$\beta c = \beta c_m \exp\left(- \frac{(\alpha - \beta)\frac{a}{1-b}\left[\left(t - \frac{t_p}{2}\right)^{1-b} - \left(\frac{t_p}{2}\right)^{1-b} \right] + \beta q_0 (t - t_p)}{h_m(\theta_s + \rho_s k)} \right) \tag{13}$$

The variation of nutrient concentration in runoff is described by Eq. (13). The runoff intensity q (L/min) can be expressed as

$$q = q_1 - aS\left(t - \frac{t_p}{2}\right)^{-b} \tag{14}$$

In Eq. (14), S is the slope area (m^2); q_1 is the total inflow (L/min).

The runoff nutrient loss of per time W (mg/min) can be expressed as

$$W = \beta c_m \exp\left(- \frac{(\alpha - \beta)\frac{a}{1-b}\left[\left(t - \frac{t_p}{2}\right)^{1-b} - \left(\frac{t_p}{2}\right)^{1-b} \right] + \beta q_0 (t - t_p)}{h_m(\theta_s + \rho_s k)} \right) \left[q_l - aS\left(t - \frac{t_p}{2}\right)^{-b} \right] \tag{15}$$

The cumulative loss of the runoff nutrient W_1 (mg) can be expressed as

$$W_1 = \beta c_m \int_{t_p}^{t} \exp\left(- \frac{(\alpha - \beta)\frac{a}{1-b}\left[\left(t - \frac{t_p}{2}\right)^{1-b} - \left(\frac{t_p}{2}\right)^{1-b} \right] + \beta q_0 (t - t_p)}{h_m(\theta_s + \rho_s k)} \right) \left[q_l - aS\left(t - \frac{t_p}{2}\right)^{-b} \right] \mathrm{d}t \tag{16}$$

The Eq. (13) describes the process of runoff variation in nutrient concentrations, and Eq. (16) describes the runoff nutrient accumulated loss. In this way, the surface runoff nutrient transfer effective mixing depth model suitable for unsaturated soil under the condition of current scour is formed.

2 Results and analysis

2.1 The variation characteristics of surface runoff

Soil nutrient transferred with surface runoff, and in order to better analyze the process of soil nutrient loss along slope land, the characteristics of runoff variation were studied. Fig. 2 shows that the runoff of Caragana and Soybean plots varies with the drainage time. As shown in Fig. 2, the runoff increases rapidly with the increase of the discharge time, and then gradually tends to be stable. This is mainly because of the extension of time, the slope cumulative infiltration gradually gets stabilized and the flow from upslope is constant. Consequently, the runoff also gradually gets stabilized. According to the characteristics of runoff and water balance, when the vegetation is Caragana, the parameters a and b in Kostiakov equation are 0.16 and 0.22; and when the vegetation is Soybean, the parameters a and b are 0.14 and 0.22.

2.2 The variation characteristics of nutrients concentration in runoff

Fig. 3 shows the change of nitrate nitrogen and water soluble phosphorusmass concentration of two vegetations in runoff. As it can be seen from Fig. 3, the general trend of the nitrate nitrogen mass concentration $C_r(t)$ and water soluble phosphorus mass concentration $C_a(t)$ changed with time similarly, and both of them decayed rapidly in the early discharge

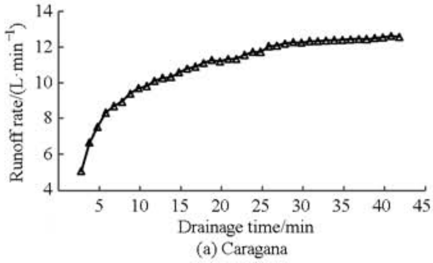

Fig. 2 Changing curves of runoff *vs* time

concentration, and then decay to a smaller mass concentration which tends to be stable with the duration of the discharge time. The incomplete-mixing model assumes that the nutrient concentration in runoff, infiltration and the effective mixing layer are not the same, but there is a linear relationship between the three (runoff, infiltration and the effective mixing layer). The nutrient concentration in runoff varies with time can not be fitted with exponential function, but the nutrient concentration in runoff exponentially decreased with 0.78 power of the time. The experimental runoff nutrient mass concentration is curved fitted as followings:

When the vegetation is Caragana

$$C_r(t) = 12.826\mathrm{e}^{-0.154[(t-0.89)^{0.78}-0.92]-0.009(t-1.787)}$$

$$(R^2 = 0.91)$$

$$C_a(t) = 9.597\mathrm{e}^{-0.190[(t-0.89)^{0.78}-0.92]-0.005(t-1.787)}$$

$$(R^2 = 0.90)$$

When the vegetation is Soybean

$$C_r(t) = 10.808\mathrm{e}^{-0.155[(t-0.755)^{0.78}-0.81]-0.006(t-1.51)}$$

$$(R^2 = 0.90)$$

$$C_a(t) = 7.089\mathrm{e}^{-0.132[(t-0.755)^{0.78}-0.81]-0.003(t-1.51)}$$

$$(R^2 = 0.92)$$

Fig. 3 showed that the fitting curve is in good agreement with the measured data, and the correlation coefficient is above 0.9. The values of the parameter α, β, h_m that obtained when fitting model parameters are shown in Tab. 1. Under two kinds of nutrient conditions, the parameter α is much larger than β, which showed that the nutrient migration pathway is mainly infiltration and the proportion of nutrient in runoff is small. The adsorption phosphorus is strong, so the water soluble phosphorus concentration in runoff should be lower than nitrate nitrogen, and it can be seen from β that the incomplete-mixing model reflects this difference. According to the assumptions of the model, it can be seen that t_0 should be less than or equal to t_p. Take the h_m that obtained from the fitting curve into Eq. (4) and t_0 is obtained. The result shows

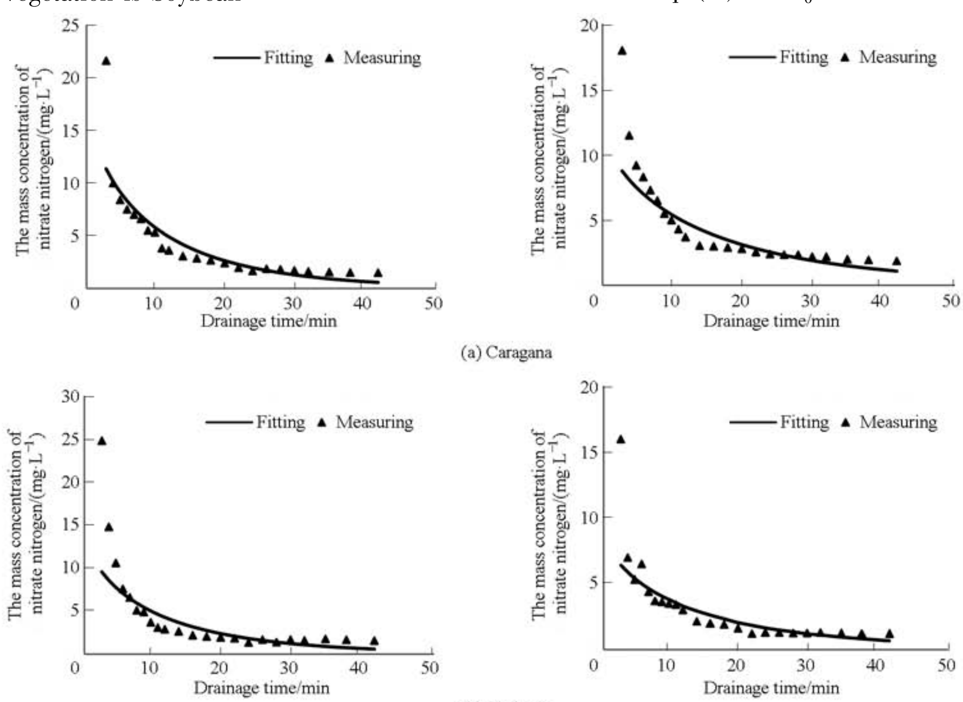

Fig. 3 Changing curves of nutrient mass concentration in runoff *vs* time

that t_0 is less than t_p under both of the two kinds of nutrient conditions, which is consistent with the model assumptions, and the obtained h_m meets the requirements of the model.

Tab. 1 Parameter values obtained under different vegetation conditions

Vegetation type	Nutrient type	α	β	h_m
Caragana	Nitrate nitrogen	0.80	0.047	0.6
	Adsorption phosphorus	0.95	0.024	0.5
Soybean	Nitrate nitrogen	0.95	0.030	0.7
	Adsorption phosphorus	0.96	0.026	0.4

2.3 Analysis of runoff nutrient cumulative loss

Soil nutrient loss process is closely related to runoff, and the runoff of soil nutrient loss is affected by twofactors, one is the amount of the runoff, and the other is the nutrient concentration in runoff. Fig. 4 shows that the cumulative loss of nitrate nitrogen and water soluble phosphorus in runoff changed with the discharge time under two planting conditions. It can be seen from the Fig. 4 that the cumulative nutrients loss in runoff increases with time. Use Eq. (16) to calculate the cumulative nutrients loss in runoff and compare with the measured data (Fig. 4), it can be found that the calculated date matches well with the measured date. When the nutrient is nitrate nitrogen, the relative error of calculated and the measured values are 6.6% (Caragana) and 5.9% (Soybean); When the nutrient is adsorption phosphorus, the relative error of calculated and the measured values are 1.1% (Caragana) and 2.3% (Soybean). Results indicated that the proposed model can accuratly predict nutrients loss.

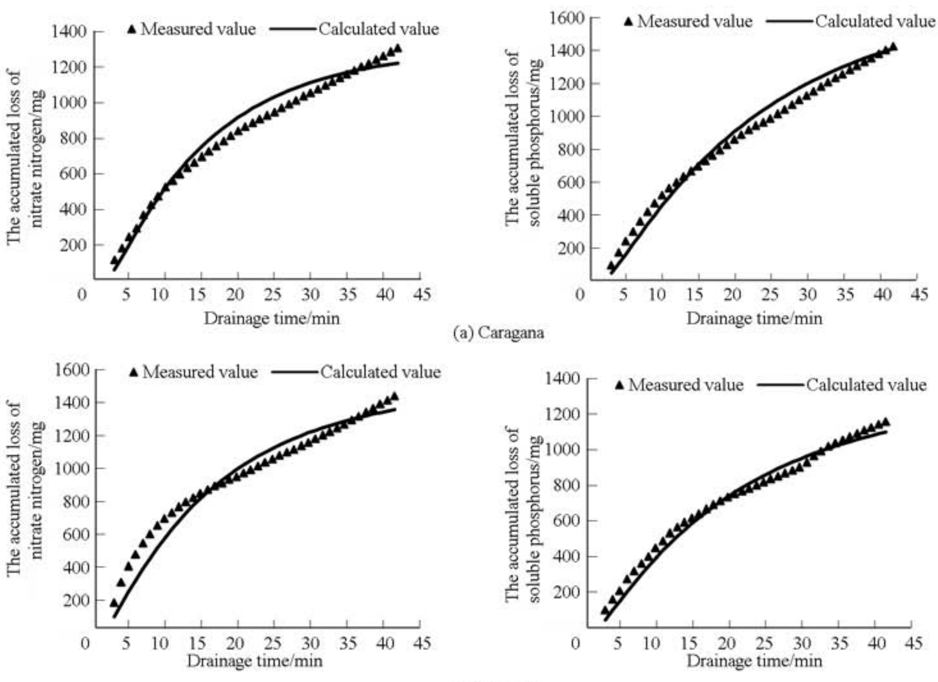

Fig. 4 Changing curves of accumulated loss of nutrient in runoff *vs* time

3 Conclusion

In this study, the effective mixing depth model of soil nutrients in the Loess Slope under the condition of water flow erosion was established, and the measured nutrients concentration was adopted to verify the model. Results showed that the proposed model can predict the variation of runoff nutrient concentration pretty good, and the correlation coefficient is above 0.9. The difference between the cumulative nutrients loss in runoff obtained from the model and measured data is very small. When the nutrient is nitrate nitrogen, the relative error of calculated and the measured values are 6.6% (Caragana) and 5.9% (Soybean); When the nutrient is adsorption phosphorus, the relative error of calculatied and the measured values are 1.1% (Caragana) and 2.3% (Soybean).

Parameters α, β, h_m in the model are related to each other, which can be obtained with the use of the process of the measured nutrient concentration in runoff and parameter optimization method. However, sometimes the obtained parameters are not unique, and the value of α and β still need to be further studied. In above, for the selection of the model, the accuracy of the calculation and convenience of obtaining parameters are needed to be considered.

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doi:10.6041/j.issn.1000-1298.2016.07.026

径流冲刷条件下坡地养分随地表径流迁移数学模型

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摘要：径流冲刷是土壤养分随地表径流流失的重要影响因素之一。为了模拟径流冲刷作用对土壤养分流失的影响,依据饱和土壤条件下有效混合深度概念,建立了径流冲刷条件下坡面土壤养分向地表径流传递的有效混合深度模型。利用试验数据对模型径流养分质量浓度变化过程的效果进行了检验,结果显示不完全混合模型在模拟径流养分质量浓度随放水时间的变化方面效果较好,决定系数均在0.9以上。通过不完全混合模型计算径流养分累计流失量,与实测数据差异很小,当养分为硝态氮时,计算值与实测值之间的相对误差分别为6.6%(柠条)和5.9%(大豆),当养分为水溶性磷时,相对误差分别为1.1%(柠条)和2.3%(大豆),说明模型的模拟精度较高。在选择模型时,应根据计算精度和参数获取方便性等情况综合确定。

关键词：径流冲刷；黄土坡面；有效混合深度；养分迁移；地表径流

中图分类号：S152.7　　**文献标识码**：A　　**文章编号**：1000-1298(2016)07-0189-07

Mathematical Model of Soil Nutrient along Surface Runoff under Water Scouring Condition

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Abstract: Overland flow is one of the important factors inducing soil nutrients loss into runoff. In order to simulate and study the effects of overland flow on soil nutrients loss, a mathematical model describing soil nutrients transport with overland flow was developed based on the concept of effective mixing depth and Kostiakvo equation. In addition, the model also makes the corresponding assumptions to simplify the infiltration process. The obtained model was denoted as the incomplete-mixing model. Field water scouring experimental data was adopted to test the applicability of the proposed model. The experiment was performed on a natural, fallowed loessial slope (38°46′~38°51′N, 110°21′~110°23′E), in 14 km west of the Shenmu Erosion and Environment Research Station of the Institute of Soil and Water Conservation, Chinese Academy of Sciences, in the Liudaogou watershed, Shaanxi Province. Field plots were established on a slope that had been fallowed for seven years. Vegetation types cover were adopted (herbaceous and crops), including caragana and soybean. The characteristics of the transport of runoff, nutrients (nitrogen and phosphorus) under different vegetation cover were discussed in this study. Results indicated that the incomplete-mixing model performed pretty well in predicting the process of nutrients transport into runoff. The correlation coefficients were larger than 0.9 for all the treatments. Moreover, little bias was observed between the measured cumulative mass and the simulated data obtained from the incomplete-mixing model. Taking nitrate nitrogen for example, the relative errors between the measured data and simulated results were 6.6% (caragana) and 5.9% (soybean). When the nutrient was soluble phosphorus, the relative errors were 1.1% (caragana) and 2.3% (soybean). For better simulation results, the calculation accuracy and simplicity in obtaining parameters should be taken into consideration during model selection. The results in this study will provide significant references for more analysis on nutrients transport into runoff with overland flow in future.

Key words: runoff scouring; loess slope; effective mixing depth; nutrient transport; surface runoff

收稿日期：2016-03-10　修回日期：2016-04-19

基金项目：国家自然科学基金重点项目(51239009)

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引言

水土流失问题是一个世界性环境问题,不仅直接导致农田养分的流失,使土地生产能力降低,而且伴随径流流失的养分和泥沙还会淤积在河道和湖泊等水源,导致水体的富营养化。在我国黄土高原地区,水土流失问题更为严重[1-3]。因此国内外学者对此进行了大量的研究,并相继提出了不同类型描述土壤养分随地表径流迁移的数学模型[4-7],其中混合深度模型由于具有明确物理意义,成为广泛应用和研究的模型[8-9]。早在 1977 年,DONIGIAN 等[10]提出模拟土壤养分随径流迁移的模拟模型,假设土壤表面有一层浅薄的混合层,降水与混合层中的土壤和土壤溶液完全混合,然后混合层中的溶解态养分按比例、以不同模式分配到该混合层的入渗水、径流水和土壤水中,混合层的厚度可通过试验数据率定得到。FRERE 等[11]认为混合层深度为 1.0 cm,并假定土壤水、径流水和入渗水完全混合,且浓度相同,即完全混合模型,然而通过模拟研究发现结果不理想。INGRAM 等[12]发现径流中的养分浓度远远低于土壤养分浓度,于是提出了不完全混合概念,认为土壤养分浓度与入渗水养分浓度相同,而与径流浓度成比例。这种模型的模拟结果相对于完全混合模型有所改善。1980 年,美国农业部在“混合层”概念的基础之上开发了非点源污染管理模型,即 CREAMS 模型。AHUJA 等[13]利用^{32}P作为示踪剂进行试验,发现表层土壤养分进入径流几率最大,而且随深度的增加,进入径流的几率呈指数衰减。因此他们提出了有效混合深度(EDI)概念,并且 EDI 是一个概化性的厚度,而非土壤的实际厚度。随后 AHUJA 等[14-15]通过试验研究发现,在较大入渗率影响下,径流养分浓度的对数值与时间呈曲线关系。这表明了对于入渗能力较大的土壤,完全混合模型是不合理的。张亚丽[16]对矿质氮素与径流作用深度进行了研究,提出了推算 EDI 的反推法和拐点法。WALLACH 等[17]通过分析不透水情况下饱和土壤养分运移特性,根据土壤养分运移方程,求出了土壤养分剖面分布,并且认为土壤养分通过质量传递方式进入径流,因而提出有效传递深度(EDT)概念。王全九[18]根据黄土地区的具体特点提出了等效径流迁移深度概念,把有效混合深度分成两部分,一部分称为等效入渗深度,另一部分称为等效径流迁移深度,并且建立了等效径流迁移深度求解方程,并根据试验资料推求了不同雨滴动能下K^+的等效径流迁移深度。由于黄土地区存在严重的水土流失,而国外大多数学者建立的指数模型并未考虑到水土流失,因此王全九等[19]根据大量的试验数据,提出了等效对流质量传递模型,通过室内模拟试验研究,发现模拟结果与实测结果较为吻合。之后王全九等[20]借助有效混合深度概念和 Philip 入渗[21]公式,推求了降雨条件下土壤养分随径流迁移有效混合深度模型。YANG 等[22]通过降雨试验研究了多种影响因素与混合层之间的关系,认为混合层深度随着降雨条件的不同而变化。

目前描述土壤养分随地表径流迁移的数学模型主要是针对降雨条件,且模型发展相对比较成熟,但是缺乏径流冲刷条件下土壤养分随地表径流迁移的数学模型。限制了在降雨条件下,同时描述雨滴打击和径流冲刷在养分迁移中的共同作用,因此需要进一步分析径流冲刷对土壤养分随地表径流迁移的影响。本文通过对坡面水流的入渗过程进行概化,结合 Kostiakov 入渗公式和养分迁移的混合深度模型建立径流冲刷条件下坡面养分随地表径流迁移的数学模型,为发展全面考虑雨滴与径流作用的坡面养分随地表径流迁移数学模型提供研究基础。

1 材料与方法

1.1 试验装置与试验材料

径流冲刷试验于 2015 年 7—9 月在中国科学院水土保持研究所神木侵蚀与环境试验站进行,试验站位于神木六道沟小流域(38°46′~38°51′N、110°21′~110°23′E),该地区水土流失严重,且暴雨、干旱等自然灾害频繁。土壤类型为砂黄土,其中粘粒、粉粒、砂粒所占比例分别为 15.19%、36.90%、47.91%(质量分数)。试验区表层(土层深度 50 cm 以内)土壤的容重变化范围在 1.26~1.41 g/cm^{3}之间,平均容重为 1.34 g/cm^3。共设置 4 个试验小区,沿小区坡面顺坡方向两侧安装挡板,防止径流流出,在小区径流出口处安装“V”形槽收集径流。坡面植被分别为柠条和大豆,每种植被设 1 个重复,种植行距为 30 cm。径流冲刷试验之前,对所有坡面小区利用水平尺进行整平,确保坡面径流同时推进,避免股流。采用定水头控制流量供水,供水设备由水窖、可动式蓄水池、水泵以及发电机等组成。试验示意图如图 1 所示。

1.2 试验方法

根据神木当地农民的施肥习惯,设计养分喷施标准为 KNO_3 60 g/m^2、KH_2PO_4 60 g/m^2,喷施之后,放置 24 h,使养分有足够的时间在表层一定深度范围分布。

试验小区坡长为 10 m,宽度为 1 m,坡度为

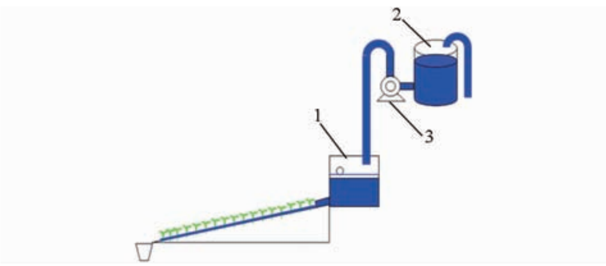

图 1　水流冲刷试验示意图

Fig. 1　Schematic diagram of water scouring experiment

1. 蓄水池　2. 水窖　3. 水泵

10.8°。上方来水流量为 21 L/min(单位面积的放水流量为 0.21 cm/min),试验前多次率定流量,当流量和均匀度均达到要求时开始径流冲刷试验,单场放水时间为 40 min,当径流冲刷试验开始后,记录坡面径流到达试验前已设定好的断面的时间,用作径流推进过程的测定,当坡面水流到达小区出口断面时,记录产流时间。产流开始后,每隔 1 min 用容量大约 18 L 的塑料桶收集径流水样,用称重法测定每个径流水样的质量(处理数据时换算为体积)。之后将各径流桶中的泥沙风干,用来测定各时段的泥沙质量。当坡面水流稳定之后,用染色剂法(高锰酸钾溶液)来测定流速。

用容量为 100 mL 的塑料瓶从收集径流的塑料桶中采集径流澄清液冷藏,测定硝态氮和水溶性磷的质量浓度;每次放水试验前后,在小区顺坡方向上、中、下坡段采集土壤剖面土样用来测定土壤养分浓度(全文养分浓度均指质量浓度),表层 10 cm 土壤按照0 ~ 1 cm、1 ~ 3 cm、3 ~ 5 cm、5 ~ 7.5 cm、7.5 ~ 10 cm 的原则采集,10 cm 以下土壤每隔 5 cm 采集一次土样,采样深度为 50 cm。测定土壤剖面含水率时,采集土样原则和上述采样原则一致,用烘干法测定土壤含水率。小区进行土壤采样和土壤水分测定之后,用小区周围土壤进行等质量回填,并进行压实,使得填充土的密实度尽量与小区内的原土壤接近。试验地土壤的饱和含水率为 40.55%(体积含水率),初始含水率分别为 9.575%(柠条)和 10.67%(大豆)(两者均为体积含水率);硝态氮和水溶性磷质量浓度用紫外分光光度计测定,采用 0.5 mol/L $NaHCO_3$浸提-钼锑抗比色法测定土壤有效磷,每次取样取3 次重复,取算术平均值作为计算结果,用该方法测得土壤表层硝态氮和有效磷的初始质量比分别为 339.12、451.05 mg/kg。通过试验测定土壤硝态氮和有效磷的等温线性吸附系数为 0.83、2.1 L/kg。数据处理用 Excel 和 Matlab 软件。

1.3　理论分析

利用考斯加可夫公式来描述径流冲刷条件下的坡面入渗过程。对于坡长较短的坡面,为了简化入渗过程,将坡面看成一个单元体,并假设整个坡面入渗从产流时间的一半开始,即入渗开始时间为 $t_p/2$,因此考斯加可夫公式可表示为

$$i = a\left(t - \frac{t_p}{2}\right)^{-b} \tag{1}$$

$$I = \frac{a}{1-b}\left(t - \frac{t_p}{2}\right)^{1-b} \tag{2}$$

式中　i——入渗率,cm/min

I——累计入渗量,cm

t——放水时间,min

t_p——产流时间,min

a——第一单位时间末土壤入渗率,cm/min

b——经验指数

径流冲刷条件下土壤养分迁移过程可以分成两部分,一是随入渗水向深层迁移,另一个是随地表径流迁移。由于传统混合深度模型大都建立在饱和土壤和降雨条件下,为了建立径流冲刷条件下非饱和土壤养分向地表径流传递数学模型,需要将土壤水分和养分迁移过程进行概化。假定坡面径流首先使混合深度达到饱和,然后土壤水分才穿过混合层,携带养分向深层迁移。有效混合深度内的养分总是均匀分布,并且进入地表径流的几率相同;无论养分以何种方式进入地表径流,都将看成以溶解态进入径流,将土壤侵蚀的综合作用看成是对有效混合深度和浓度比例系数的影响;在整个放水期间混合层内养分浓度与入渗水向深层携带的养分及随地表径流迁移的养分浓度呈固定比例关系[20]。

如果土壤的饱和含水率为 θ_s(单位:cm^3/cm^3),初始含水率为 θ_i(单位:cm^3/cm^3),有效混合层深度为 h_m(单位:cm),则饱和有效混合深度内土壤需补充的含水量 w_0(单位:cm)为

$$w_0 = (\theta_s - \theta_i)h_m \tag{3}$$

则由

$$I_0 = w_0 \tag{4}$$

有

$$\frac{a}{1-b}\left(t_0 - \frac{t_p}{2}\right)^{1-b} = (\theta_s - \theta_i)h_m \tag{5}$$

则饱和有效混合深度内土壤需要的时间 t_0为

$$t_0 = \left[\frac{1-b}{a}(\theta_s - \theta_i)h_m\right]^{\frac{1}{1-b}} + \frac{t_p}{2} \tag{6}$$

式中　I_0——时间为 t_0时的累计入渗量,cm

如果 $t_p < t_0$,则应调整有效混合深度,使两者相等。如果 $t_p > t_0$,则按照上述假设计算产流时刻混合层内养分浓度。如果将养分吸附过程看成是等温线性吸附过程,则有

$$c_s = kc_0 \tag{7}$$

式中　c_s——吸附在土壤颗粒上的养分质量比,mg/kg

k——等温线性吸附系数,L/kg

c_0——土壤溶液养分浓度,mg/L

根据 AHUJA[4] 提出的有效混合深度概念,并认为有效混合深度内养分浓度与入渗水携带养分浓度以及径流养分浓度成比例,假定时间达到产流时刻 t_p 时,混合层开始向地表径流传递养分,此时有效混合深度内养分浓度(单位:mg/L)可表示为

$$c_m=\frac{h_m(\theta_s+\rho_s k)c_i}{\alpha\left[\frac{a}{1-b}\left(\frac{t_p}{2}\right)^{1-b}-(\theta_s-\theta_i)h_m\right]+h_m(\theta_s+\rho_s k)} \tag{8}$$

式中 ρ_s——土壤容重,g/cm³

c_i——土壤养分初始浓度换算成饱和状态的养分浓度

α——入渗水养分浓度与有效混合深度内养分浓度的比值

产流之后,有效混合深度内的养分一部分随地表径流迁移,另一部分随入渗水向深层迁移,因此有效混合深度内养分随时间变化可表示为

$$\frac{\mathrm{d}ch_m(\theta_s+\rho_s k)}{\mathrm{d}t}=-\alpha ic-\beta qc \tag{9}$$

式中 β——径流养分浓度与有效混合深度内养分浓度的比值

q——单位面积径流强度,cm/min

c——产流后任意时刻有效混合深度养分质量浓度,mg/L

对式(9)积分得

$$c=c_m\exp\left(-\frac{\alpha\int_{t_p}^{t}i\mathrm{d}t+\beta\int_{t_p}^{t}q\mathrm{d}t}{h_m(\theta_s+\rho_s k)}\right) \tag{10}$$

径流养分浓度可表示为

$$\beta c=\beta c_m\exp\left(-\frac{\alpha\int_{t_p}^{t}i\mathrm{d}t+\beta\int_{t_p}^{t}q\mathrm{d}t}{h_m(\theta_s+\rho_s k)}\right) \tag{11}$$

式中第 1 个积分项表示累计入渗量,第 2 个积分项表示累计径流量,利用考斯加可夫公式计算累计入渗量,用入流总量减去累计入渗量则得到累计径流量,表示为

$$\beta c=\beta c_m\exp\left(-\frac{\alpha\frac{a}{1-b}\left[\left(t-\frac{t_p}{2}\right)^{1-b}-\left(\frac{t_p}{2}\right)^{1-b}\right]+\beta q_0(t-t_p)-\beta\frac{a}{1-b}\left[\left(t-\frac{t_p}{2}\right)^{1-b}-\left(\frac{t_p}{2}\right)^{1-b}\right]}{h_m(\theta_s+\rho_s k)}\right) \tag{12}$$

式中 q_0——单位面积的入流量,cm/min

进一步简化,得到不完全混合模型

$$\beta c=\beta c_m\cdot\exp\left(-\frac{(\alpha-\beta)\frac{a}{1-b}\left[\left(t-\frac{t_p}{2}\right)^{1-b}-\left(\frac{t_p}{2}\right)^{1-b}\right]+\beta q_0(t-t_p)}{h_m(\theta_s+\rho_s k)}\right) \tag{13}$$

式(13)描述了径流养分浓度变化过程。由于养分流失量是径流量和养分浓度的乘积,径流强度 q(单位:L/min)可以表示为

$$q=q_1-aS\left(t-\frac{t_p}{2}\right)^{-b} \tag{14}$$

式中 S——坡面面积,m²

q_1 的单位为 L/min,则单位时间内的径流养分流失量 W(单位:mg/min)表示为

$$W=\beta c_m\cdot\exp\left(-\frac{(\alpha-\beta)\frac{a}{1-b}\left[\left(t-\frac{t_p}{2}\right)^{1-b}-\left(\frac{t_p}{2}\right)^{1-b}\right]+\beta q_0(t-t_p)}{h_m(\theta_s+\rho_s k)}\right)\cdot\left[q_1-aS\left(t-\frac{t_p}{2}\right)^{-b}\right] \tag{15}$$

径流养分累计流失量 W_1(单位:mg)表示为

$$W_1=\beta c_m\cdot\int_{t_p}^{t}\exp\left(-\frac{(\alpha-\beta)\frac{a}{1-b}\left[\left(t-\frac{t_p}{2}\right)^{1-b}-\left(\frac{t_p}{2}\right)^{1-b}\right]+\beta q_0(t-t_p)}{h_m(\theta_s+\rho_s k)}\right)\cdot\left[q_1-aS\left(t-\frac{t_p}{2}\right)^{-b}\right]\mathrm{d}t \tag{16}$$

式(13)描述了径流养分浓度变化过程,式(16)描述了径流养分累计流失量,由此形成了径流冲刷条件下适应于非饱和土壤的地表径流养分迁移有效混合深度模型。

2 结果与分析

2.1 坡面径流量变化特征

径流是坡面养分流失的载体,深入研究径流变化特征,才能更好地分析坡面土壤养分流失过程。图 2 为 10 m 坡长条件下柠条和大豆试验小区各自径流量随放水时间的变化情况。由图 2 可知,放水初期径流量随着放水时间增加迅速递增,随着放水时间的持续,径流量逐步趋于稳定。这主要是因为随着放水时间的延长,坡面累计入渗量逐步趋于稳定,上方来水流量一定,因此径流量也逐步趋于稳定。根据水流特征和水量平衡,当植被为柠条时,获得考斯加可夫公式中参数 a、b 的值分别为 0.16、0.22,当植被为大豆时,获得参数 a、b 的值分别为 0.14、0.22。

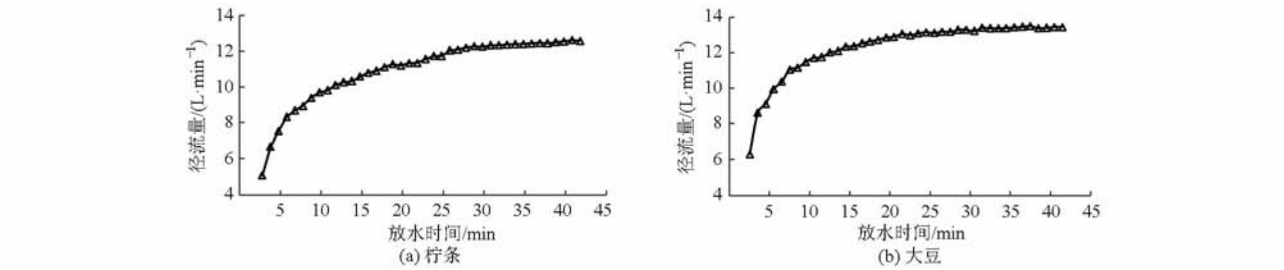

图 2　径流量随放水时间的变化曲线

Fig. 2　Changing curves of runoff *vs* time

2.2　径流养分浓度变化特征

图 3 为 2 种植被条件下径流硝态氮和水溶性磷质量浓度随时间的变化过程。从图中可以看出，硝态氮质量浓度 $C_r(t)$ 和水溶性磷质量浓度 $C_a(t)$ 随时间的总体变化趋势相似，两者均是在放水初期质量浓度迅速衰减，而后随着放水时间的持续，质量浓度衰减到一个较小的趋于稳定的浓度。不完全混合模型假定径流养分浓度、入渗水养分质量浓度和有效混合层养分质量浓度不相同，但三者之间呈线性关系。径流养分质量浓度与放水时间不呈指数关系，而是与放水时间的 0.78 次方呈指数关系。对实测径流养分质量浓度进行曲线拟合，结果如下：

植被为柠条时

$$C_r(t)=12.826\mathrm{e}^{-0.154[(t-0.89)^{0.78}-0.92]-0.009(t-1.787)}$$

$$(R^2=0.91)$$

$$C_a(t)=9.597\mathrm{e}^{-0.190[(t-0.89)^{0.78}-0.92]-0.005(t-1.787)}$$

$$(R^2=0.90)$$

植被为大豆时

$$C_r(t)=10.808\mathrm{e}^{-0.155[(t-0.755)^{0.78}-0.81]-0.006(t-1.51)}$$

$$(R^2=0.90)$$

$$C_a(t)=7.089\mathrm{e}^{-0.132[(t-0.755)^{0.78}-0.81]-0.003(t-1.51)}$$

$$(R^2=0.92)$$

从图 3 可以看出拟合曲线与实测数据较吻合，决定系数均在 0.9 以上。拟合模型参数时所获得参数 α、β、h_m 的值见表 1。2 种养分条件下，参数 α 均远大于参数 β，说明养分的迁移途径主要是以入渗为主，而进入径流的养分占养分总量的比例很小。磷的吸附性较强，因此进入径流的水溶性磷质量浓度应低于硝态氮，从 β 可以看出不完全混合模型体现出了此差异。根据模型的假设可知，t_0 应小于或等于 t_p，将拟合所获得的 h_m 代入式(6)求得 t_0，结果显示 2 种养分条件下的 t_0 均小于 t_p，符合模型的假设，所获得的 h_m 满足模型需求。

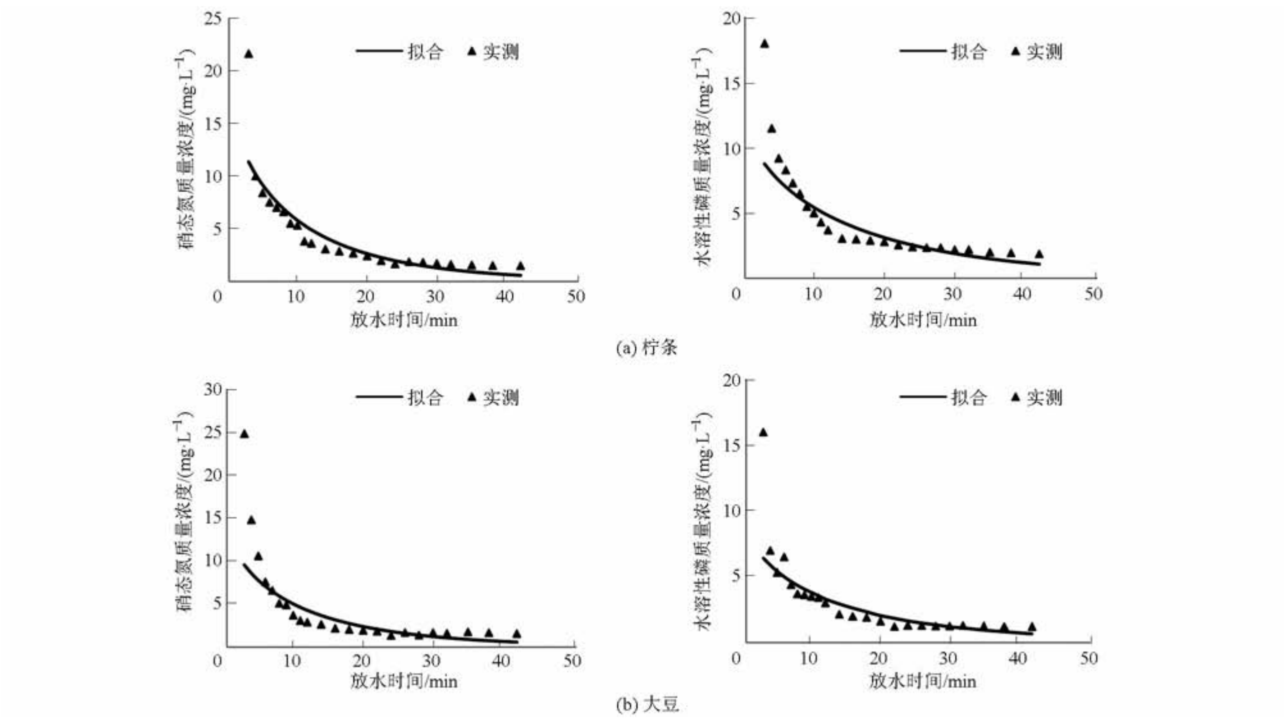

图 3　径流养分浓度随放水时间的变化曲线

Fig. 3　Changing curves of nutrient concentration in runoff *vs* time

表 1 不同植被条件下获得的参数值

Tab. 1 Parameter values obtained under different vegetation conditions

植被类型	养分类型	α	β	h_m
柠条	硝态氮	0.80	0.047	0.6
	水溶性磷	0.95	0.024	0.5
大豆	硝态氮	0.95	0.030	0.7
	水溶性磷	0.96	0.026	0.4

2.3 径流养分累计流失量分析

坡面土壤养分流失过程与径流关系密切，地表径流中土壤养分流失量受 2 个方面的影响，一是径流量，二是径流中的养分浓度。图 4 为 2 种植被条件下径流硝态氮和水溶性磷累计流失量随放水时间的变化情况。从图中可以看出，径流养分累计流失量随着放水时间的延长逐渐增大。利用式(16)计算得到径流养分累计流失量，与实测数据进行对比(图 4)，发现两者吻合效果较好，当养分为硝态氮时，计算值与实测值之间的相对误差分别为 6.6%(柠条)和 5.9%(大豆)，养分为水溶性磷时，相对误差分别为 1.1%(柠条)和 2.3%(大豆)，说明模型的模拟精度较高。

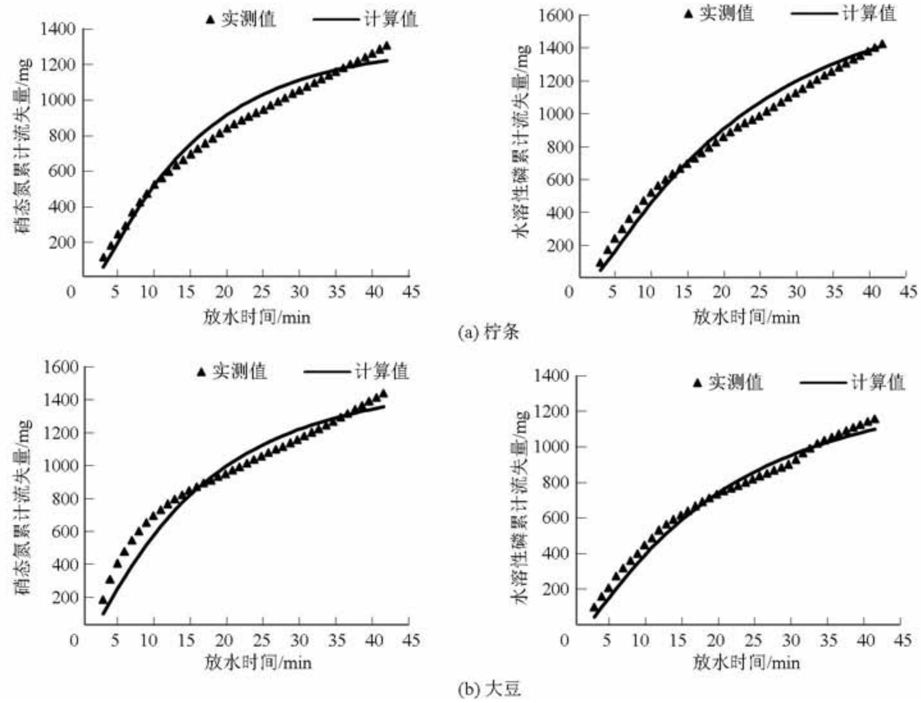

图 4 径流养分累计流失量随放水时间变化曲线

Fig. 4 Changing curves of accumulated loss of nutrient in runoff *vs* time

3 结束语

建立了水流冲刷条件下黄土坡面土壤养分随地表径流迁移的有效混合深度模型，并利用试验资料对模型径流养分质量浓度变化过程的效果进行了检验，结果显示不完全混合模型在模拟径流养分质量浓度随放水时间的变化过程方面具有很好的效果，决定系数均在 0.9 以上。通过不完全混合模型计算径流养分累计流失量，与实测数据差异很小，当养分为硝态氮时，计算结果与实测值之间的相对误差分别为 6.6%(柠条)和 5.9%(大豆)，养分为水溶性磷时，相对误差分别为 1.1%(柠条)和 2.3%(大豆)，说明模型的模拟精度较高。不完全混合模型包含 α、β、h_m 3 个参数，而且这 3 个参数相互关联，需要利用实测径流养分浓度过程和参数优选方法获得，这样易于造成参数的不唯一性，同时 α 和 β 取值问题仍需要进一步研究。因此在模型选择方面，应根据计算精度和参数获取方便性等情况综合确定。

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